A true test for VaR

The three classic approaches for measuring portfolio Var do not compare like with like, argues *Richard Sage*. Here he presents a test portfolio to highlight the differences between calculation methods

Textbooks, lectures, and articles* classically state that there are three approaches for calculating value-at-risk (VaR) on a portfolio of financial assets and liabilities: variance-covariance, historical simulation and Monte Carlo simulation. But these methods do not compare like with like – they mix two separate dimensions.

This may be clearer if we put the jargon to one side for the moment and consider transportation. If we needed a vehicle and were offered the choice of 'bicycle', 'petrol', or 'diesel', most of us would respond: "What type of petrol or diesel vehicle?" If we were then offered a car or van with either a petrol or diesel engine, our first choice would probably be between bicycle, car and van, and only after that would we consider the type of fuel.

Similarly in the world of VaR, we first need to choose what it is we are going to calculate: volatilities, complete values or movements in values? We can calculate the latter two using either historical or Monte Carlo simulation, just as both cars and vans – but not bicycles – can be fitted with either petrol or diesel engines. No approach for calculating VaR gives a 'right' number, as we cannot predict the future accurately, but the best estimate can be useful.

Table 1: **xxxxx**

CALCULATED	PRICE INPUTS		
	Current only	Historical simulation	Monte Carlo simulation
Volatilities	1	NA	NA
Movements in values	NA	2	3
Complete values	NA	4	5

Source: author

Variance-covariance

Under the variance-covariance approach, we assume that returns on the portfolio are normally distributed with a mean of zero. The shape of a normal distribution depends only on its standard deviation, so the size of the range of values – which will include *x*% of the values – is a function of only the standard deviation and the proportion of values to be included. For example, if we seek the 95% VaR, then the tables of the normal distribution tell us that the boundary above which 95 % of values fall is 1.645 times the standard deviation.

Standard deviations are also called volatilities. Therefore, using the variance-covariance approach, we combine the volatilities (standard deviations) of the individual instruments within the portfolio and the correlations between them – either calculated from historical data or selected by the operator using his experience and judgment – to calculate an overall standard deviation (volatility), then multiply it by the relevant scaling factor.

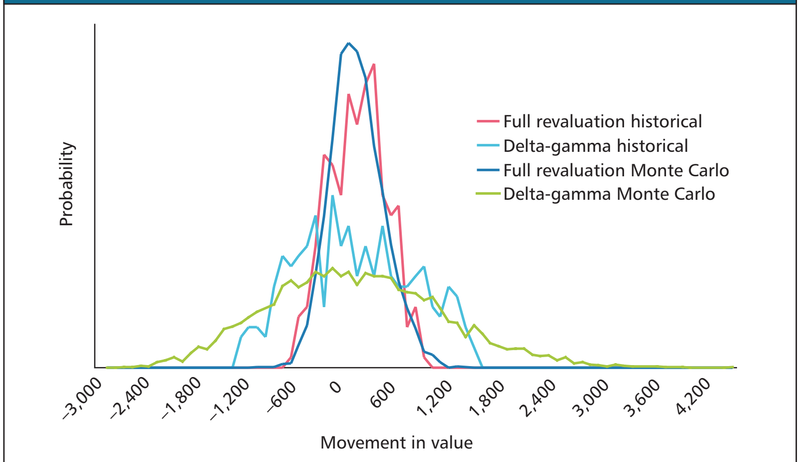
Simulation

The various simulation approaches do not explicitly assume any specific distribution of returns. They all involve considering what would be the effect on the value of the portfolio of each of a fairly large number, *n*, of scenarios and taking the relevant percentile by simply sorting the outcomes of the n scenarios and taking say the (95% * *n*)th.

There are two independent choices to make. One concerns how we generate the input prices. We can either model price movements by a random stochastic process or by following historical data. Doing the latter obviously limits the number of scenarios to the size of our dataset of past movements – say 2,500 if we were to use 10 years of data – but includes situations that really happened, however unlikely they may have seemed before their occurrence. Using randomly generated movements, we can run as many scenarios as we want, although in most situations, running more than 10,000 has a minimal effect on the final answer.

The other choice is whether we revalue each instrument in each scenario or use an approximation for the change in value. If we revalue each instrument, then the movement in value in each scenario is the difference between the value in the base case and the value in the relevant scenario. The recalculation

Distributions for the five approaches



[1] *For example, 'How to Spot a VaR cheat',* EPRM *May 2003, page 42.*

Table 2: xxxxx

Approach	What is calculated	Simulation approach	Also known as	Requires (in addition to units, values and volatilities of each position)	Specific assumptions additional to those inherent in the input requirements
1	Volatilities	–	Parametric approach	Correlations between price Variance-covariance	Movement in valuation of portfolio has normal distribution movements
2	Movements in values	Historical using delta (and gamma)	Delta (gamma) –	Deltas (and gammas) normal different from the past	Value of portfolio has linear or second-order relation to input prices. The future will be not totally
3	Movements in values using delta (and gamma)	Monte Carlo	Delta (gamma) –	Stochastic process. Deltas (and gammas). Correlations between price movements.	Value of portfolio has linear or second-order relation to input prices
4	Full revaluation	Historical	–	Fast revaluations	The future will be not totally different from the past
5	Full revaluation	Monte Carlo	–	Stochastic process. Very fast revaluations. Correlations between price movements.	

Source: author

of each instrument in each scenario requires a lot of computational resource when instruments such as options are involved. For this approach, the VaR calculation engine must have models for every instrument in the portfolio.

One approximation that would reduce the size of the computational resource required is to calculate the movement in value for each instrument directly by multiplying the first-order sensitivities (partial derivatives) of the value to a number of input parameters by specific changes in those input parameters. This is known as the delta-normal approach. It is clearly not sufficient for instruments whose values are not linearly dependent on the value of the input parameters.

We can improve the approximation by adding second-order sensitivities, resulting in the delta-gamma approach.

Using these approximations, the VaR calculation engine does not require models for every instrument in the portfolio (see table 1). However, the results may be misleading for instruments where the delta – and gamma – sensitivities do not fully explain the movements in value over the range of the movements in inputs for all the different scenarios.

We could, therefore, visualise the overall choice as being between one of five approaches. Table 2 compares these approaches, by looking at the assumptions they make and their requirements. The differences between the approaches can be significant.

As an example, consider a test portfolio consisting of long positions in four equity options (see table 3).

We took historical prices for the underlying for a period of 250 days. They gave volatilities of 26.79% for A, 19.32% for B and 22.35% for C. The risk-free rate was 5%, with a volatility of 0.18%.

The result of the variance-covariance approach is by definition a perfect normal distribution, so we usually calculate just a single number. The other four approaches give empirical distributions.

The results for the distributions are shown in the graph. Table 4 shows the results for the 95% VaR.

These results suggest that in at least some situations the choice between historical simulation and Monte Carlo simulation is less important than the choice between calculating complete values and only movements in values (delta-gamma).

The complete-revaluation approach requires more computational resources, but the theory supporting it is more robust than for the movements-in-value approach – which is explicitly an approximation to the complete-revaluation approach. EPRM

Richard Sage is financial controller at global commodity trading group Trafigura in London.
email: ras.trafigura@zsalya.co.uk

Table 3: xxxxx

Underlying equity	C | P	Strike	Days to expiry	Units	Spot value of underlying
A	Call	100	70	500	103
A	Put	105	70	700	103
B	Call	80	102	240	107
C	Put	60	191	(900)	70

Source: author

Table 4: xxxxx

CALCULATED	PRICE INPUTS		
	Current only	Historical simulation	Monte Carlo simulation
Volatilities	(776)	NA	NA
Movements in values	NA	(1,026)	(1,572)
Complete values	NA	(496)	(440)

Source: author